AC/14.06.2021/RS1

SIES College of Arts, Science & Commerce

RISE WITH EDUCATION

NAAC REACCREDITED "A" GRADE, CGPA 3.51/4.00

(AUTONOMOUS)

SIES College of Arts, Science and Commerce
(Autonomous)
Affiliated to University of Mumbai

Program B.Sc.

Subject: MATHEMATICS

Class: FYBSc

Syllabus revised in June 2021

Choice Based Credit System (CBCS)
with effect from the academic year 2021-22

CONTENTS

1. Preamble

Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. It is imperative that the content of the undergraduate syllabi of Mathematics should support other branches of science such as Physics, Chemistry, Statistics, Computer Science, Biotechnology. This syllabus of F. Y. B. Sc. Semesters I and II have been designed to provide learners sufficient knowledge and skills enabling them to undertake further studies in Mathematics and its allied areas. There are two courses for 'Calculus'(Paper1) spread over two semesters. Calculus is applied and needed in every conceivable branch of science. The courses 'Algebra I and Discrete Mathematics' (Paper2) develop mathematical reasoning and logical thinking and have applications in science and technology.

2. Learning Objectives of the courses:

1. To develop critical thinking, reasoning and logical skills of the learners
2. To improve learners' analytical and problem solving skills
3. To take the learners from simple to difficult and from concrete to abstract
4. To equip learners with a deeper understanding of abstract theory and concepts
5. To improve learners' capacity to communicate mathematical/logical ideas in writing.

3. Programme Outcomes

SIES has integrated the Learning Outcome Based Curriculum Framework in the syllabi of all the programmes since the academic year 2021-22. Upon completing the B.Sc. Mathematics Programme, the students are expected to develop the following abilities and skills:

I. **Solving Complex Problems:**

Applying the knowledge of various courses learned under a program with an ability to break down complex problems into simple components, by designing processes required for problem solving.

II. **Critical Thinking and reasoning ability:**

Exhibits ability to understand abstract concepts, analyze, and apply them in problem solving. Ability to formulate and develop logical arguments, developing the ability to think with different perspectives and ideas. (Skills necessary for progression to higher education and research.)

III. **Research Aptitude:**

Acquiring the ability to explore and gain knowledge in independent ways through reading assignments, problem solving assignments, projects, seminars, presentations.

IV. **Information and Digital literacy**:

Equip to select, apply appropriate tools and techniques, resources through electronic media for the purpose of visualizing mathematical objects, geometrical interpretations, coding, and analyzing data.

V. **Sound Disciplinary knowledge:**

Demonstrate comprehensive knowledge and understanding of the fundamental concepts and theories of mathematics; apply them to interdisciplinary areas of study.

VI. **Communicating Mathematical Ideas:**

Organize and deliver mathematical ideas through effective written, verbal, graphical/virtual communications.

4. Course structure with minimum credits and Lectures/ Week

SEMESTER I				
Paper1 : CALCULUS I (Theory Course)				
Course Code	UNIT	TOPICS	Credits	L /week
SIUSMAT11	I	Real Number System	2	3L
	II	Limit and Continuity of real valued functions		
	III	First order First degree Differential equations		
Paper2: ALGEBRA I (Theory Course)				
Course Code	UNIT	TOPICS	Credits	L /week
SIUSMAT12	I	Integers and divisibility	2	3L
	II	Functions and Equivalence relations		
	III	Polynomials		
PRACTICALS				
Course Code	TOPICS		Credits	P/week
SIUSMATP1	Practicals based on courses SIUSMAT11 & SIUSMAT12		2	1P(=2L)

SEMESTER II				
Paper1: CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/week
SIUSMAT21	I	Sequences of real numbers and sequential continuity, properties of continuous functions	2	3L
	II	Differentiation of real valued functions of 1 variable		
	III	Applications of derivatives		
Paper2: DISCRETE MATHEMATICS				
Course Code	UNIT	TOPICS	Credits	L/week
SIUSMAT22	I	Preliminary Counting	2	3L
	II	Advanced Counting		
	III	Permutations and Recurrence relation		
PRACTICALS				
Course Code	TOPICS		Credits	P/week
SIUSMATP2	Practicals based on courses SIUSMAT21 & SIUSMAT22		2	1P(=2L)

5. Consolidated Syllabus for semester I & II with Course Outcomes

SEMESTER I

<table>
<tr><td colspan="3">Course Code: SIUSMAT11
Course Name: CALCULUS I
(Paper I)</td></tr>
<tr><td colspan="3">Expected Course Outcomes
On completion of this course, students will be able to
1. State definitions, propositions and prove important results based on Supremum, Infimum , bounded sets, properties and inequalities of real numbers, limit, continuity, order and degree of an ODE and its various types
2. Apply various properties, results and inequalities to solve problems on intervals, neighbourhoods, boundedness and to solve ODEs
3. Determine continuity at a point or on intervals and distinguish between the types of discontinuities at a point, Identify bounded and unbounded sets, Identify the type of ODEs and solve it using appropriate methods</td></tr>
<tr><td>Pre-requisites:</td><td colspan="2">Introduction to sets $N,\ Q,\ Z,\ R$</td></tr>
<tr><td>Unit I</td><td>Real Number System</td><td>15 Lectures</td></tr>
<tr><td></td><td colspan="2">● Real Numbers: basic properties of real numbers. order properties of R, Law of Trichotomy, Absolute value function and its properties, AM-GM inequality, Cauchy-Schwarz inequality
● Intervals and neighbourhoods in R , Hausdorff property
● Bounded sets in the set of Real numbers, Supremum and infimum, Basic results: Continuum property (l.u.b. Axiom-statement) and consequences, Archimedean property and its applications.</td></tr>
<tr><td>Unit II</td><td>Limit and Continuity of real valued functions</td><td>15 Lectures</td></tr>
<tr><td></td><td colspan="2">● Real valued Functions of one variable and Graphs: Domain, Range. Graph of a function. Examples : Constant function, Identity function, Absolute value, Step function, Floor and Ceiling functions, Trigonometric functions, Linear and Quadratic functions and their graphs. Graphs of functions such as x^3, $\frac{1}{x}$, $\frac{1}{x^2}$, log(x), a^x and e^x.
● ε - δ definition of limit of a function,
Evaluation of limit of simple functions using the definition, Uniqueness of limit if it exists, Algebra of limits (with proof), Limit of composite function
● Sandwich theorem, Left hand, Right hand limits,
Non-existence of limit
● Infinite limits and Limits at infinity.</td></tr>
</table>

	• Continuity of a real valued function at a point in terms of Limits Continuity of a real valued function on a set in terms of Limits, examples, Continuity of a real valued function at end points of domain • Algebra of continuous functions • Discontinuous functions, examples of removable and essential discontinuity,	
Unit III	First order First degree Differential equations	15 Lectures
	• Review of Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and nonlinear ODE. Solution of homogeneous and non homogeneous differential equations of first order and first degree. Notion of partial derivatives. • Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $M\,dx + N\,dy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non exact equations. • Reduction of order : (i) If the differential equation does not contain only the original function y, that is, equations of Type $F(x, y', y'') = 0$. (ii) If the differential equation does not contain the independent variable x that is, equations of Type $F(y, y', y'') = 0$.	

References for SIUSMAT11 (Paper I)

1. R. G. Bartle- D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
2. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
3. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd.
4. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982
5. G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 1972. .

6.A. R. Forsyth, A Treatise on Differential Equations, MacMillan and Co.,1956

7.James Stewart, Calculus, Third Edition, Brooks/ cole Publishing Company,1994

8. Ajitkumar, S. Kumaresan, A Basic Course in Real Analysis, CRC press, 2014

Online Resources

1. https://openstax.org/details/books/calculus-volume-1
2. https://archive.org/details/Calculus10thEditionH.Anton

<table>
<tr><td colspan="3">Course Code: SIUSMAT12
Course Name: ALGEBRA I
(Paper II)</td></tr>
<tr><td colspan="3">Expected Course Outcomes
On completion of this course, students will be able to
1. State definitions of divisibility of integers, GCD, prime, congruence, function and its different types, binary operation, relation and its different types, polynomial, root of a polynomial. State the well ordering property, induction theorems, fundamental theorem of Arithmetic, theorems associated to roots of various polynomials, State and prove results based on divisibility, primes, congruences, bijectivity of functions, binary operations, partitions and equivalence relations, roots and irreducibility of polynomials
2. Apply various results to find GCD, prove propositions based on induction theorems, solve problems based on congruences, check bijectivity of functions, find roots of a polynomial, GCD of polynomials
3. Identify invertible functions, binary operations, partitions and equivalence relations, irreducible polynomials, factors of a polynomial, multiplicity of a root.</td></tr>
<tr><td>Pre-requisites:</td><td colspan="2">Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations.</td></tr>
<tr><td>Unit I</td><td>Integers and divisibility</td><td>15 Lectures</td></tr>
<tr><td></td><td colspan="2">● Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of Well-Ordering Principle.
● Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two non-zero integers, basic properties of g.c.d. such as existence and uniqueness and that the g.c.d. of $a, b \in Z$ can be expressed as $ma + nb,\ m, n \in Z$, Euclidean algorithm.
● Primes, Euclid's lemma, Fundamental Theorem of arithmetic, The set of primes is infinite, there are arbitrarily large gaps between primes, there exists infinitely many primes of the form $4n - 1$ or of the form $6n - 1$.
● Congruence, definition and elementary properties, Results about linear congruence equations. Examples.</td></tr>
<tr><td>Unit II</td><td>Functions and Equivalence relations</td><td>15 Lectures</td></tr>
<tr><td></td><td colspan="2">● Definition of a function, domain, co-domain and range of a function, composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(B)$ for a function f; Injective, surjective, bijective functions; Composite of injective, surjective, bijective functions when defined; invertible functions, bijective functions are invertible and conversely; examples of functions including constant, identity, projection, inclusion;</td></tr>
</table>

	● Binary operation as a function, properties, examples. ● Equivalence relation, Equivalence classes, properties such as two equivalence classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa. Congruence is an equivalence relation on Z, Residue classes and partition of Z.	
Unit III	Polynomials	15 Lectures
	● Definition of a polynomial, polynomials over the field F where F = Q, R or C, Algebra of polynomials, degree of polynomial, basic properties, ● Division algorithm in F[X] (without proof), and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, ● Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem, A polynomial of degree n has at most n roots , Complex non real roots of a polynomial in R[X] occur in conjugate pairs, Statements of Fundamental Theorem of Algebra, A polynomial of degree n in C[X] has exactly n complex roots counted with multiplicity, A non constant polynomial in R[X] can be expressed as a product of linear and quadratic factors in R[X], necessary condition for a rational number p/q to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is an irrational number where p is a prime number ● Irreducible polynomials in $Q[X]$,Unique Factorisation Theorem. Examples.	

References for SIUSMAT12 (Paper II)
1. Larry J. Gerstein.(2012). *Introduction to Mathematical Structures and Proofs,* Springer-Verlag, New York 2. J. P. Tremblay & R. Manohar.(1974).*Discrete Mathematical Structures with Applications to Computer Science,* McGraw Hill 3. David M. Burton.(2015). *Elementary Number Theory,* McGraw Hill Education (India) Private Ltd. 4. Norman L. Biggs.(1989).*Discrete Mathematics*, Clarendon Press, Oxford 5. I. Niven and S. Zuckerman.(1972).*Introduction to the theory of numbers,* Wiley Eastern 6. G. Birkoff and S. Maclane. (1965).*A Survey of Modern Algebra,*Mac Millan, New York 7. N. S. Gopalkrishnan.(2013).*University Algebra*, Ne Age International Ltd 8. I .N. Herstein, , John Wiley.(2006).*Topics in Algebra* 9. P. B. Bhattacharya S. K. Jain and S. R. Nagpaul.(1994). *Basic Abstract Algebra*, New Age International 10. Kenneth Rosen.(1999). *Discrete Mathematics and its applications*, Mc-Graw Hill International Edition, Mathematics Series. 11. Ajit Kumar & S Kumaresan.(2018).*Foundation Course in Mathematics,* Narosa

Course Code: SIUSMATP1 Course Name: Practicals based on SIUSMAT11 AND SIUSMAT12	
	Expected Course Outcomes On completion of this course, students will be able to 1. Apply various definitions, results and methods learnt in two theory courses to solve problems. 2. Test validity of mathematical statements using results and constructing appropriate examples
	Practicals in SIUSMAT11
1	A. Application based examples on Properties of real numbers, Archimedean property, intervals, neighbourhood, Hausdorff Property. B. Identifying Bounded and Unbounded sets, computing the Infimum and Supremum of a set, Consequences of LUB Axiom.
2	A. Evaluation of limits B. Continuity
3	A. Solving exact and non-exact, linear, reducible to linear differential equations. B. Reduction of order of Differential Equations, Applications of Differential Equations.
	Practicals in SIUSMAT12
1	A. Mathematical induction, Division Algorithm and Euclidean algorithm B. Primes and the Fundamental Theorem of Arithmetic, Congruences
2	A. Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions B. Equivalence relations and Partitions
3	A. Division Algorithm and GCD of polynomials, Factor Theorem B. Relation between roots and coefficients of polynomials, factorization

SEMESTER II

Course Code: SIUSMAT21 Course Name: CALCULUS II (PAPER I)		
Expected Course Outcomes On completion of this course, students will be able to 1. State the definitions of convergent, divergent, oscillating , bounded and monotone sequences, derivatives and related terms, Plot graphs of standard functions and comment on continuity; State and prove results on convergence and boundedness of sequences, differentiability, Mean Value theorems and extreme values of a function. 2. Apply various results to check boundedness, convergence of sequences, Apply the notions of continuity and differentiability to algebraic and transcendental functions to solve problems and to compute higher order derivatives 3. Identify critical points and classify into maxima, minima saddle points, classify sequences and other real valued functions based upon their properties		
Pre-requisites	Review of concepts of limits.	
Unit I	Sequences	15 Lectures
	• Definition of a sequence and examples, Definition of convergent and divergent sequences. Limit of sequence, uniqueness of limit if it exists. Simple examples such as seq(1/n) , where convergence is checked using ϵ-n_0 definition • Sandwich theorem, Algebra of convergent sequences , Examples. • Monotonic and Bounded sequences: Definition of bounded sequences. Every convergent sequence is bounded. Monotone sequences and the Monotone convergence theorem. Examples. • Subsequences, Bolzano Weierstrass Theorem • Cauchy sequences, relation between Cauchy and convergent sequences, Examples • Limits of some special sequences like (1+1/n)^n, (n)^(1/n)	
Unit II	Continuity and Differentiation	15 Lectures
	• Sequential continuity and related examples • Statements of properties of continuous functions on closed intervals such as Intermediate Value Property, Attainment of bounds, Location of roots and examples.	

	• Definition of Derivatives of a real valued function of one variable at a point and on an open set • Left / Right Derivatives, Examples of differentiable and non- differentiable functions • Geometric/ Physical Interpretation of derivative • Differentiable functions are continuous but not conversely • Algebra of differentiable functions, Chain Rule for derivative of a composite function (statement only) • Derivative of inverse functions, Implicit differentiation (only examples) • Higher order derivatives of some standard functions, Leibnitz rule.	
Unit III	Applications of derivatives	15 Lectures
	• The Mean Value Theorems: Rolle's theorem, Lagrange's mean value theorem. examples and applications, Cauchy's mean value theorems, examples and applications, • Taylor's theorem with Lagrange's form of remainder (without proof). Taylor's polynomial and applications. • Indeterminate forms:L'Hospital's Rule (without proof) Examples. • Applications of first and second derivatives: Monotone increasing and decreasing function, examples, using derivatives. Concave, Convex functions, Points of Inflection Definition of local maximum and local minimum, First derivative test for extrema, Necessary condition for extrema, Stationary pointsSecond derivative test for extrema, examples, Global maxima and minima	

References for Paper I
1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.. 2. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd. 3. G. B. Thomas, R.L. Finney ,Calculus and analytic geometry, 3rd Edition onwards. 4. Ghorpade,Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2000 5. Ajitkumar, S. Kumaresan, A Basic Course in Real Analysis, CRC press, 2014
Online Resources
1. https://openstax.org/details/books/calculus-volume-1 2. https://archive.org/details/Calculus10thEditionH.Anton

Course Code: SIUSMAT22
Course Name: DISCRETE MATHEMATICS
(Paper 2)

Expected Course Outcomes

On completion of this course, students will be able to:

1. State definitions of countable sets, Stirling number of second kind, derangements, permutations, recurrence relations. State Pigeonhole principle, multinomial theorem, inclusion and exclusion principle, State and prove results based on countability of sets, permutations, combinations, Stirling numbers, identities based on multinomial theorem
2. Solve problems based on counting principles, pigeonhole principles, multinomial theorem, Inclusion & Exclusion principle, derangements, recurrence relations
3. Classify sets based on countability, Identify recurrence relations as homogeneous/non-homogeneous, types of permutations

Pre-requisites:	Principle of Mathematical Induction, Permutations, Combinations	
Unit I	Preliminary Counting	15 Lectures

- Finite and infinite sets, countable and uncountable sets, examples such as $N, Z, N \times N, Q, (0, 1), R$
- Addition and multiplication Principle, counting sets of pairs, two way counting.
- Stirling numbers of the second kind. Simple recursion formulae satisfied by $S(n, k)$ for $k = 1, ..., n$
- Pigeonhole principle and its strong form, examples, its applications to geometry.

Unit II	Advanced Counting	15 Lectures

- Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
- Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs:

i. $\sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$

ii. $\sum_{i=r}^{n}\binom{i}{r} = \binom{n+1}{r+1}$

iii. $\sum_{i=0}^{k}\binom{k}{i}^2 = \binom{2k}{k}$

iv. $\sum_{i=0}^{n}\binom{n}{i} = 2^n$

- Non-negative and positive integral solutions of equation $x_1 + x_2 + ... + x_k = n$
- Principle of inclusion and exclusion, its applications, derangements, explicit formula for d_n, Euler's function $\varphi(n)$.

Unit III	Permutations and Recurrence relation	15 Lectures
	• Permutation of objects, S_n, composition of permutations results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, signature of a permutation, even and odd permutations, cardinality of Sn, An. • Recurrence Relations, definition of homogeneous, non-homogeneous, linear, non-linear recurrence relation, obtaining recurrence relations of Tower of Hanoi, Fibonacci sequence, etc. in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.	

References for Paper II

1. Norman Biggs: Discrete Mathematics, Oxford University Press.
2. Richard Brualdi: Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy: Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications,TataMcGrawHills.
5. Schaum's Outline series: Discrete mathematics,
6. Applied Combinatorics: Allen Tucker, John Wiley and Sons.

Course Code: SIUSMATP2 Course Name: Practicals based on SIUSMAT21 and SIUSMAT22	
	Expected Course Outcomes On completion of this course, students will be able to 1. Apply various definitions, results and methods learnt in two theory courses to solve problems. 2. Test validity of mathematical statements using results and constructing appropriate examples
	Practicals in SIUSMAT21
1	A. Convergence of sequences. B. Sequential continuity and Properties of continuous functions.
2	A. Differentiability, relation between differentiability and continuity. B. Higher order derivatives, Leibnitz theorem.
3	A. Mean value theorems and its applications. B. Extreme values, increasing and decreasing functions.
	Practicals in SIUSMAT22
1	A. Counting principles, Two way counting. B. Stirling numbers of the second kind, the Pigeon hole principle.
2	A. Multinomial theorem, identities, B. permutation and combination of multi-set. Inclusion-Exclusion principle. Euler phi function.
3	A. Composition of permutations, signature of permutation, inverse of permutation. B. Recurrence relation.

6. Teaching Pattern for semester I & II

1. Three lectures per week per course in each semester.
2. One practical per week per batch based on all courses. (2 lectures per batch per week)
3. Minimum 4 practicals to be conducted per batch per course in each semester.
4. Conduct of Practicals: The Practicals shall be conducted in batches formed as per the University circular. The Practical session shall consist of discussion between the teacher and the students in which students should participate actively. The students should maintain a journal for practicals which should be submitted for checking regularly and at the end of the semester.

7. Scheme of Evaluation for Semesters I & II

The performance of the learners shall be evaluated in three ways:
(a) Continuous Internal Assessment of 40 marks in each course in each semester.
(b) Semester End Examinations of 60 marks in each course at the end of each semester.
(c) A combined Practical exam of 100 marks for all the three courses at the end of each semester.

(a) Continuous Internal Assessment (40 marks) :

Sr No	Evaluation type	Marks
1	One class test	20
2	Reading Assignment and Presentation/ Written assignment/ Project (Any one method chosen by teacher)	20
	Total	40

(b) Semester End examination (60 marks) :

Duration: 2 hours
Question Paper Pattern: All units will have equal weightage in the question paper of each theory course.

(c) Semester End Practical Examination (100 marks):

At the end of each semester, one practical examination of 2 hours duration and 100 marks shall be conducted for the courses. Students will have to submit a certified journal.

Semester 1: Semester End Practical Examination		
Sr. No.	**Evaluation type**	**Marks (out of)**
1	Part A : Questions from SIUSMAT11	40
2	Part B : Questions from SIUSMAT12	40
3	Certified Journals	20
4	Total	100

Semester 2: Semester End Practical Examination		
Sr. No.	**Evaluation type**	**Marks (out of)**
1	Part A : Questions from SIUSMAT21	40
2	Part B : Questions from SIUSMAT22	40
3	Certified Journals	20
4	Total	100

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